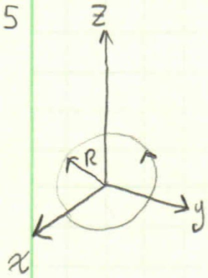
Hinkle, Zachary | EM Griffiths | 08/09/18 | 1/1

5.35



a) $\vec{m} = I\vec{a} = I \pi R^2 \hat{z}$

b) $A_{dip} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi r^2} I \pi R^2 \hat{z} \times (x\hat{x} + y\hat{y} + z\hat{z}) \frac{1}{r}$

$$A_{dip} = \frac{\mu_0 I}{4 \cdot r^3} R^2 (y\hat{y} - x\hat{x})$$

$$\vec{B}_{dip} = \vec{\nabla} \times \vec{A}_{dip} = -\frac{\partial A_y}{\partial z}\hat{x} + \frac{\partial A_x}{\partial z}\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$$

$$\frac{\partial}{\partial x}\frac{1}{r^3} = \frac{\partial}{\partial x}(x^2+y^2+z^2)^{-3/2} = \frac{-(3/2)(2x)}{(x^2+y^2+z^2)^{5/2}} = -\frac{3x}{r^5}$$

Likewise

$$\frac{\partial}{\partial y}\frac{1}{r^3} = -\frac{3y}{r^5} \quad \text{and} \quad \frac{\partial}{\partial z}\frac{1}{r^3} = -\frac{3z}{r^5}$$

so

$$\vec{B}_{dip} = \frac{\mu_0 I R^2}{4}\left[-\frac{\partial}{\partial z}\left(-\frac{x}{r^2}\right)\hat{x} + \frac{\partial}{\partial z}\left(-\frac{y}{r^2}\right)\hat{y} + \left\{\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) - \frac{\partial}{\partial y}\left(-\frac{y}{r^3}\right)\right\}\hat{z}\right]$$

$$\vec{B}_{dip} = \frac{\mu_0 I R^2}{4}\left[-\frac{3xz}{r^5}\hat{x} + \frac{3yz}{r^5}\hat{y} + \left\{\frac{1}{r^3} - \frac{3x^2}{r^5} - \left(\frac{-1}{r^3} + \frac{3y^2}{r^5}\right)\right\}\hat{z}\right]$$

$$\vec{B}_{dip} = \frac{\mu_0 I R^2}{4r^5}\left[-3xz\hat{x} + 3yz\hat{y} + \{2r^2 - 3(x^2+y^2)\}\hat{z}\right]$$

c) for points on the z-axis, $x=y=0$, and $\vec{B}_{dip}$ becomes

$\frac{\mu_0 I R^2}{4r^5}[2r^2]\hat{z} = \frac{\mu_0 I R^2}{2z^{3/2}}\hat{z}$ and $\left(\sqrt{R^2+z^2}\right)^3 \approx z^{3/2}$ for $z >> R$,

So this equation is consistent w/ eq. 41