5.28 eq. 65: $\vec{A}(\vec{r}') = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|} d\tau'$

a) $\vec{\nabla}\cdot\vec{A}(\vec{r}') = \frac{\mu_0}{4\pi}\vec{\nabla}\cdot\int\frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}d\tau'$

$= \frac{\mu_0}{4\pi}\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)\int\frac{J_x'\hat{x} + J_y'\hat{y} + J_z'\hat{z}}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}dx\,dy\,dz$

$= \frac{\mu_0}{4\pi}\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)\left[\int\frac{J_x'\hat{x}}{|\vec{r}-\vec{r}'|}dx'dy'dz' + \int\frac{J_y'\hat{y}}{|\vec{r}-\vec{r}'|}dx'dy'dz' + \dots\right]$

$= \int\frac{\partial}{\partial x}\left(\frac{J_x(\vec{r}')}{\sqrt{(x-x')^2+\dots}}dx'dy'dz'\right) + \dots$

$= \int\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{(x-x')^2+\dots}}\right)J_x(\vec{r}')\,dx'dy'dz' + \dots$

$= -\int\frac{\partial}{\partial x'}\left(\frac{1}{\sqrt{(x-x')^2+\dots}}\right)J_x(\vec{r}')\,dx'dy'dz' + \dots$

$= \int\frac{\frac{\partial}{\partial x'}(J_x(\vec{r}'))}{\sqrt{(x-x')^2+\dots}}dx'dy'dz' - \left.\frac{J_x(\vec{r}')}{\sqrt{(x-x')^2+\dots}}\right|_A + \dots$

We can choose a boundary, A, s.t. all 3 boundary terms will be 0. This is accomplished by picking a volume large enough that at the surface $\vec{J}=0$.

$= \int\frac{\partial}{\partial x'}\left[J_x(\vec{r}')\right]\frac{1}{|\vec{r}-\vec{r}'|}d\tau' + \int\frac{\partial}{\partial y'}\left[J_y(\vec{r}')\right]\frac{1}{|\vec{r}-\vec{r}'|}d\tau' + \int\frac{\partial}{\partial z'}\left[J_z(\vec{r}')\right]\frac{1}{|\vec{r}-\vec{r}'|}d\tau'$

$= \int\frac{\vec{\nabla}'\cdot\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}d\tau' = -\int\left(\frac{\partial\rho}{\partial t}\right)\frac{d\tau'}{r} = 0$ since $\frac{\partial\rho}{\partial t}=0$ in magnetostatics