5.26

a) $\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$; $\vec{B} = \vec{\nabla}\times\vec{A}$

$$\vec{\nabla}\times\vec{A} = \left[\frac{1}{s}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sA_\phi) - \frac{\partial A_s}{\partial\phi}\right]\hat{z}$$

So

$$\frac{\partial A_z}{\partial\phi} = s\frac{\partial A_\phi}{\partial z} \; ; \quad \frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s} = \frac{\mu_0 I}{2\pi s} \; ; \quad A_\phi + s\frac{\partial A_\phi}{\partial s} = \frac{\partial A_s}{\partial\phi}$$

One solution is $A_z = A_\phi = 0$, $A_s = \frac{\mu_0 I}{2\pi s} z$

i.e. $\vec{A} = \frac{\mu_0 I}{2\pi}\left(\frac{z}{s}\right)\hat{s}$

Clearly $\vec{\nabla}\times\vec{A} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$

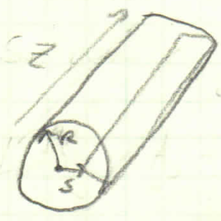
and

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{s}\frac{\partial}{\partial s}(sA_s) + \frac{1}{s}\cancelto{0}{\frac{\partial A_\phi}{\partial\phi}} + \cancelto{0}{\frac{\partial A_z}{\partial z}} = \frac{1}{s}\frac{\partial}{\partial s}\left(\frac{\mu_0 I z}{2\pi}\right) = 0$$

b) Inside the wire,

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = \mu_0 I\left(\frac{s}{R}\right)^2$; $\oint \vec{B}\cdot d\vec{l} = 2\pi s B \Rightarrow$

$\vec{B} = \frac{\mu_0 I}{2\pi R^2} s\hat{\phi}$. $\oint \vec{A}\cdot d\vec{l} = \int \vec{B}\cdot d\vec{a}$



$$\oint \vec{A}\cdot d\vec{l} = \pm z\left[A_z(R) - A_z(s)\right]$$

$$\int \vec{B}\cdot d\vec{l} = \int_0^z \int_s^R \frac{\mu_0 I}{2\pi R^2} s' ds' dz' = \frac{\mu_0 I}{4\pi R^2} z(R^2 - s^2)$$

$$\Rightarrow \pm z\left[A_z(R) - A_z(s)\right] = \frac{\mu_0 I}{4\pi R^2} z(R^2 - s^2)$$

so $A_z(s) = \pm\frac{\mu_0 I}{4\pi R^2} s^2$ since we require $\frac{\partial A_z}{\partial s} - \frac{\partial A_z}{\partial s} = \frac{\mu_0 I}{2\pi R^2} s$

$\frac{\partial A_z}{\partial s} = \pm\frac{\mu_0 I}{2\pi R} s$ implies $A_z(s) = -\frac{\mu_0 I}{4\pi R^2} s^2$

From which it is clear that $\vec{\nabla} \times \vec{A} = \vec{B}$ is satisfied if we simply let $A_s = A_\phi = 0$.

Since then

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{s}\frac{\partial}{\partial s}(s \cancel{A_s}^{0}) + \frac{1}{s}\frac{\partial \cancel{A_\phi}^{0}}{\partial \phi} + \frac{\partial A_z}{\partial z} = \frac{\partial}{\partial z}\left(-\frac{\mu_0 I}{4\pi R^2} s^2\right) = 0.$$