Hinkle, Zachary | EM Griffiths | 08/06/18 | 1/1

5-25 $\vec{\nabla}\cdot\vec{A} = -\frac{1}{2}\left[\vec{\nabla}\cdot(\vec{r}\times\vec{B})\right]$

by vector identity # 6,

$$\vec{\nabla}\cdot(\vec{r}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{r}) - \vec{r}\cdot(\vec{\nabla}\times\vec{B})$$

but $\vec{\nabla}\times\vec{r} = 0$ and $\vec{\nabla}\times\vec{B} = 0$ b/c $\vec{B}$ is uniform

so

$$\vec{\nabla}\cdot\vec{A} = -\frac{1}{2}(0) = 0.$$

$$\vec{\nabla}\times\vec{B} = -\frac{1}{2}\left[\vec{\nabla}\times(\vec{r}\times\vec{B})\right]$$

by the 8th vector identity,

$$\vec{\nabla}\times(\vec{r}\times\vec{B}) = (\vec{B}\cdot\vec{\nabla})\vec{r} - (\vec{r}\cdot\vec{\nabla})\vec{B} + \vec{r}(\vec{\nabla}\cdot\vec{B}) - \vec{B}(\vec{\nabla}\cdot\vec{r})$$

$$= \vec{B} - 0 + 0 - \vec{B}(3) = -2\vec{B}$$

so $\vec{\nabla}\times\vec{A} = -\frac{1}{2}[-2\vec{B}] = \vec{B}$ as expected.

Note, the 2nd and 3rd terms are zero b/c $\vec{B}$ is uniform & therefore any derivative is zero.

The first term is just $\vec{B}$ b/c $(\vec{B}\cdot\vec{\nabla}) = B_x\frac{\partial}{\partial x} + B_y\frac{\partial}{\partial y} + B_z\frac{\partial}{\partial z}$

So $(\vec{B}\cdot\vec{\nabla})\vec{r} = \left(B_x\frac{\partial x}{\partial x} + B_y\frac{\partial y}{\partial x} + B_z\frac{\partial z}{\partial x}\right)\hat{x} + \left(B_y\frac{\partial y}{\partial x}\ \ldots\right)\hat{y} + \ldots$

Which is obviously $B_x\hat{x} + B_y\hat{y} + B_z\hat{z} = \vec{B}$.

The final ~~2~~ term is because $\vec{\nabla}\cdot\vec{r} = 3$. Just think about it.