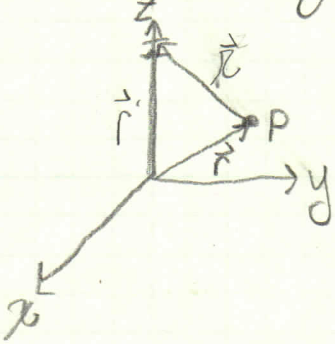
Hinkle, Zachary | EM Griffiths | 08/06/18 | 1/1

5.23



$$\vec{r}' = z'\hat{z}$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} = r\cos\phi\sin\theta\hat{x} + r$$

$$\vec{\mathscr{r}} = -x\hat{x} - y\hat{y} + (z' - z)\hat{z}$$

$$\mathscr{r}^2 = x^2 + y^2 + (z' - z)^2\hat{z}$$

$$\mathscr{r} = \sqrt{r^2 + (z')^2 - 2zz'}$$

$$= \sqrt{(z')^2 + 2zz' + r^2}$$

$$d\vec{l}' = dz\hat{z}$$

$$A = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}}{\mathscr{r}}$$

$$\vec{A} = \frac{\mu_0 I}{4\pi}\int_{z'=z_1}^{z'=z_2}\frac{dz'\,\hat{z}}{\sqrt{r^2 + (z')^2 - 2z'z}} =$$

This resembles integral (39) in the table:

$$\int\frac{dx}{\sqrt{a^2x^2 + bx + c}} = \frac{1}{\sqrt{a}}\ln\left|2ax + b + 2\sqrt{a(ax^2 + bx + c)}\right|$$

here, $x = z'$, $a = 1$, $b = -2z$, and $c = r^2$, so

$$\vec{A}\left(\frac{4\pi}{\mu_0 I}\right) = \ln\left|2z' - 2z + 2\sqrt{(z')^2 - 2zz' + r^2}\right|_{z_1}^{z_2}\hat{z}$$

$$\vec{A}\left(\frac{4\pi}{\mu_0 I}\right) = \ln\left|\frac{2(z_2 - z) + 2\sqrt{z_2^2 - 2z_2 z + r^2}}{2(z_1 - z) + 2\sqrt{z_1^2 - z_1 z + r^2}}\right|\hat{z}$$

$$\Rightarrow \vec{A} = \frac{\mu_0 I}{4\pi}\ln\left(\frac{z_2 - z + \sqrt{z_2^2 + 2z_2 z + r^2}}{z_1 + z + \sqrt{z_1^2 - 2z_1 z + r^2}}\right)\hat{z}$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \left(\frac{\partial V_z}{\partial y}\right)\hat{x} - \left(\frac{\partial V_z}{\partial x}\right)\hat{y}$$