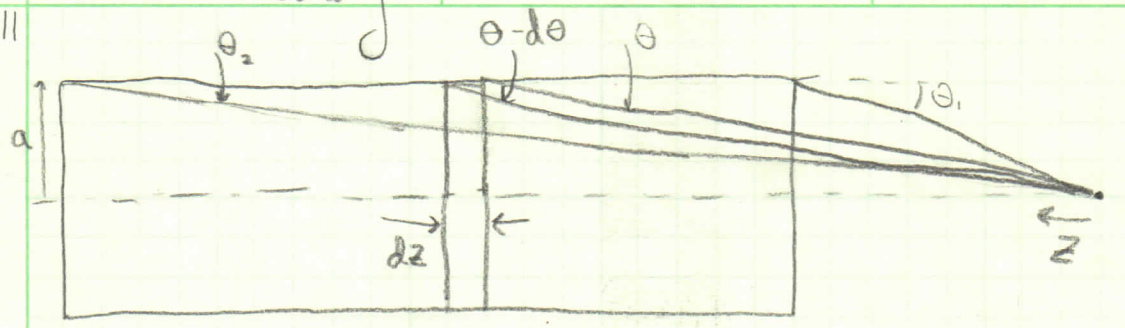
5.11



$$\sin\theta = \frac{a}{(a^2+z^2)^{1/2}} \; ; \quad \sin(\theta-d\theta) = \frac{a}{\sqrt{a^2+(z+dz)^2}}$$

$$\cos\theta = \frac{z}{(a^2+z^2)^{1/2}}$$

$$\sin x = \sin x_0 + (x-x_0)\frac{d}{dx}\sin x\Big|_{x=x_0} + \dots$$

Choosing $x_0 = \theta$, $x = \theta - d\theta$

$$\sin(\theta-d\theta) \approx \sin\theta + (\theta - d\theta - \theta)\cos\theta = \sin\theta - \cos\theta\, d\theta$$

$$a^2(a^2+x^2)^{-1/2} = a(a^2+x_0^2)^{-1/2} + (x-x_0)\frac{d}{dx}a(a^2+x^2)^{-1/2}\Big|_{x=x_0} + \dots$$

$$\approx a(a^2+x_0^2)^{-1/2} + (x_0-x)\frac{-a x_0}{(a^2+x_0^2)^{-3/2}}$$

Choose $x = z+dz$, $x_0 = z$

$$a(a^2+(z+dz)^2)^{-1/2} \approx a(a^2+z^2)^{-1/2} + dz\,\frac{az}{(a^2+z^2)^{-3/2}}$$

Since $\sin(\theta-d\theta) = a^2(a^2+(z+dz)^2)^{-1/2}$,

$$\sin\theta - \cos\theta\, d\theta = \frac{a}{\sqrt{a^2+z^2}} + \frac{az}{(a^2+z^2)^{+3/2}}dz = \sin\theta + \frac{1}{a}\sin^2\theta\cos\theta\, dz$$

$$\Rightarrow -\cos\theta\, d\theta = \frac{1}{a}\sin^2\theta\cos\theta\, dz \Rightarrow -\frac{a\,d\theta}{\sin^2\theta} = dz$$

Now, in the element dz,

$$dB = \frac{\mu_0}{2}(nI\,dz)\frac{a^2}{(a^2+z^2)^{3/2}} = \frac{n\mu_0 I}{2a}\sin^3\theta\, dz = \frac{n\mu_0 I}{2a}\sin^3\theta\left(\frac{-a}{\sin^2\theta}\right)d\theta$$

$$dB = -\frac{n\mu_0 I}{2}\sin\theta\, d\theta \Rightarrow B = -\int_{\theta_1}^{\theta_2}\frac{n\mu_0 I}{2}\sin\theta\, d\theta = \frac{n\mu_0 I}{2}(\cos\theta_2 - \cos\theta_1)$$