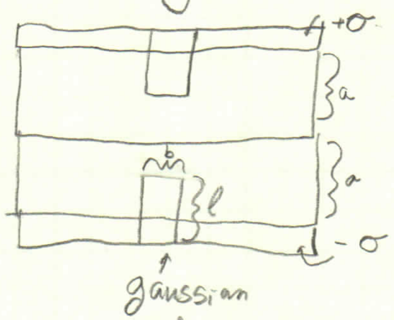
4.18 a)



surfaces: Regular rectangular prisms of length l and face width b

$$b^2 D_+ = b^2\sigma \Rightarrow \vec{D}_+ = \sigma(-\hat{z})$$

$$b^2 D_- = b^2(-\sigma) \Rightarrow \vec{D}_- = -\sigma\hat{z}$$

b) $\vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{\vec{D}}{\varepsilon_0\varepsilon_r}$; $(\varepsilon_r)_{top} = 2$, $(\varepsilon_r)_{bottom} = 1.5$

$$\Rightarrow \vec{E}_+ = \frac{\vec{D}}{2\varepsilon_0} = -\frac{\sigma}{2\varepsilon_0}\hat{z} \quad , \quad \vec{E}_- = -\frac{\sigma}{1.5\varepsilon_0}\hat{z}$$

c) $\vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E} = \frac{(\varepsilon_r-1)\vec{D}}{\varepsilon_r} = \left(1-\frac{1}{\varepsilon_r}\right)\vec{D} \Rightarrow \vec{P}_+ = \left(1-\frac{1}{2}\right)(-\sigma\hat{z}) = -\frac{\sigma}{2}\hat{z}$

$$\vec{P}_- = \left(1-\frac{2}{3}\right)(-\sigma\hat{z}) = -\frac{\sigma}{3}\hat{z}$$

d) $V = \int_{top}^{bottom} \vec{E}\cdot d\vec{l}$ Choose the path $d\vec{l} = dz\hat{z} \Rightarrow V = \int_{top}^{center} \vec{E}_+\cdot(dz\hat{z}) + \int_{center}^{bottom} \vec{E}_-\cdot(dz\hat{z})$ Choose bottom as $z=0$

$$V = -\int_{2a}^{a}\frac{\sigma}{2\varepsilon_0}dz - \int_{a}^{0}\frac{\sigma}{1.5\varepsilon_0}dz = \frac{\sigma a}{2\varepsilon_0} + \frac{\sigma a}{1.5\varepsilon_0} = \frac{\sigma a}{\varepsilon_0}\left(\frac{1}{2}+\frac{2}{3}\right) = \frac{7\sigma a}{6\varepsilon_0}$$

e) Clearly $-\vec{\nabla}\cdot\vec{P}_\pm = 0$ so $\rho_f = 0$

$$\sigma_{top} = \vec{P}_+\cdot\hat{z} = -\frac{\sigma}{2} \qquad \sigma_{bottom} = \vec{P}_-\cdot(-\hat{z}) = \frac{\sigma}{3}$$

$$\sigma_{center} = \vec{P}_+\cdot(-\hat{z}) + \vec{P}_-\cdot(\hat{z}) = \frac{\sigma}{2} - \frac{\sigma}{3} = \frac{\sigma}{6}$$

Note that $\sigma_{top} + \sigma_{center} + \sigma_{bottom} = \sigma\left(-\frac{1}{2} + \left[\frac{1}{2}-\frac{1}{3}\right] + \frac{1}{3}\right) = 0$ as expected

f) Now we may apply Gauss' Law directly using the $\vec{E}$-field:

$$b^2 \vec{E}_+ = b^2 \frac{\sigma_{enc}}{\varepsilon_0} \Rightarrow E = \frac{1}{\varepsilon_0}(\sigma + \sigma_{top}) = \frac{\sigma}{2\varepsilon_0} \Rightarrow \vec{E}_+ = -\frac{\sigma}{2\varepsilon_0}\hat{z}$$

$$b^2 E_- = b^2 \frac{\sigma_{enc}}{\varepsilon_0} \Rightarrow E = \frac{1}{\varepsilon_0}(-\sigma + \sigma_{bottom}) = -\frac{2\sigma}{3\varepsilon_0} \rightarrow \vec{E}_- = -\frac{2\sigma}{3\varepsilon_0}\hat{z}$$

which is the same as in part b.