

4.8

The energy of a dipole, \vec{p}_2 , in the field of a different dipole, \vec{E}_1 , is

$$U = -\vec{p}_2 \cdot \vec{E}_1, \quad \vec{E}_1 = \frac{1}{4\pi\epsilon_0 r^3} [3(\vec{p}_1 \cdot \hat{r})\hat{r} - \vec{p}_1]$$

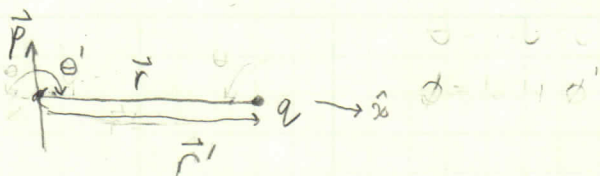
$$\text{Thus } U = -\vec{p}_2 \cdot \left(\frac{1}{4\pi\epsilon_0 r^3} [3(\vec{p}_1 \cdot \hat{r})\hat{r} - \vec{p}_1] \right)$$

$$= \frac{1}{4\pi\epsilon_0 r^3} [\vec{p}_1 \cdot \vec{p}_2 - 3(\vec{p}_1 \cdot \hat{r})(\vec{p}_2 \cdot \hat{r})]$$

4.9

The force on the charge is given by $\vec{F}_q = q\vec{E}_{\text{dip}}$

$$\vec{E}_{\text{dip}} = \frac{p}{4\pi\epsilon_0 r^3} (2\cos\theta\hat{r}' + \sin\theta\hat{\theta}') \quad \text{here } \vec{r}' = -\vec{r} \Rightarrow \hat{r}' = -\hat{r}$$



$$\vec{F}_q = q\vec{E}_{\text{dip}} = \frac{qp}{4\pi\epsilon_0 r^3} (-2\cos\theta\hat{r} + \sin\theta\hat{\theta})$$

The force on the dipole must be equal and opposite by

Newton's 3rd law, so

$$\vec{F}_p = -\vec{F}_q = \frac{qp}{4\pi\epsilon_0 r^3} (2\cos\theta\hat{r} - \sin\theta\hat{\theta})$$

$$\vec{F}_p = \frac{qp}{4\pi\epsilon_0 r^3} (2\cos\theta\hat{r} - \sin\theta\hat{\theta})$$

$$= r\cos\theta\hat{r} - r\sin\theta\hat{\theta}$$