

3.28

$$V(\vec{r}) \approx \frac{1}{4\pi\epsilon_0} \left[ \frac{1}{r} \int \lambda(\vec{r}') dl' + \frac{1}{r^2} \int r' \cos \alpha \lambda(\vec{r}') dl' + \frac{1}{r^3} \int (r')^2 \left( \frac{3}{2} \cos \alpha - \frac{1}{2} \right) \lambda(\vec{r}') dl' \right]$$

Here  $\lambda(\vec{r}') = \lambda_0$ ,  $dl' = R d\phi$ , and assuming the point is on the z-axis,  $\alpha = \frac{\pi}{2}$  and  $\therefore \cos \alpha = 0$

$$\int \lambda(\vec{r}') dl' = \lambda \int dl' = 2\pi R \lambda$$

$$\int r' \cos \alpha \lambda(\vec{r}') dl' = 0$$

$$\int (r')^2 \left( \frac{3}{2} \cos \alpha - \frac{1}{2} \right) \lambda(\vec{r}') dl' = -\frac{1}{2} \int (r')^2 \lambda(\vec{r}') dl'$$

$$-\frac{1}{2} \int (r')^2 \lambda(\vec{r}') dl' = -\frac{1}{2} \int R^2 \cdot R d\phi = -\pi \lambda R^3$$

$$V(z) = \frac{1}{4\pi\epsilon_0} \left[ \frac{1}{z} (2\pi R \lambda) + \frac{1}{z^3} (-\pi \lambda R^3) \right] = \frac{1}{4\pi\epsilon_0} \left[ 2\pi \lambda \left( \frac{R}{z} \right) - \pi \lambda \left( \frac{R}{z} \right)^3 \right]$$

$$= \frac{\pi \lambda R}{4\pi\epsilon_0 z^2} \left[ 2 - \left( \frac{R}{z} \right)^2 \right]$$