3.27. To 1^{st} order

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r}\int \rho(\vec{r}\,')\,d\tau' + \frac{1}{r^2}\int r'\cos\alpha\,\rho(\vec{r}\,')\,d\tau'\right].$$

Here we have $\rho(r',\theta) = k\frac{R}{r'^2}(R-2r')\sin\theta$ and
$d\tau' = r'^2\sin\theta\,dr'\,d\theta\,d\phi$ and
$\cos\alpha = \cos\theta$

$$\int \rho(\vec{r}\,')\,d\tau = 2\pi k R\int \frac{1}{r'^2}(R-2r')\,r'^2\sin^2\theta\,dr'd\theta$$

$$= 2\pi k R\int (R-2r')\sin^2\theta\,d\theta\,dr'$$

$$\int_0^\pi \sin^2\theta\,d\theta = \frac{1}{2}\int_0^\pi(1-\cos 2\theta)\,d\theta = \frac{1}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^\pi = \frac{\pi}{2}$$

$$\int \rho(\vec{r}\,')\,d\tau' = \pi^2 R k\int (R-2r')\,dr' = \pi^2 R k\left[Rr' - r'^2\right]_0^R = 0$$

$$\int r'\cos\alpha\,\rho(\vec{r}\,')\,d\tau' = kR\int r'\cos\theta\,\frac{1}{r'^2}(R-2r')\sin^2\theta\, r'^2\,dr\,d\theta\,d\phi$$

$$2\pi k R\int r'(R-2r')\cos\theta\sin^2\theta\,dr'\,d\theta.$$

$$\int r'(R-2r')\,dr' = \int (r'R - 2r'^2)\,dr' = \left[\frac{R}{2}r'^2 - \frac{2}{3}r'^3\right]_0^R = \frac{R^3}{2} - \frac{2R^3}{3}$$

$$= \frac{3R^3 - 4R^3}{6} = \frac{-1}{6}R^3$$

let $u = \sin\theta \Rightarrow du = \cos\theta\,d\theta$

$$\int_0^\pi \cos\theta\sin^2\theta\,d\theta = \int_0^0 u^2\,du = 0$$

Evidently, to 1^{st} order $V(\vec{r}) \approx 0$. Let's take the next term as well.

3.27 $\int r'^2\left(\frac{3}{2}\cos^2\theta-\frac{1}{2}\right)\rho(\vec{r}')\,d\tau' = 2\pi kR\int\left(\frac{3}{2}\cos^2\theta-\frac{1}{2}\right)(R-2r')r'^2\sin^2\theta\,dr\,d\theta$

$$\Rightarrow \int\left(\frac{3}{2}\cos^2\theta-\frac{1}{2}\right)(R-2r')(r'^2)\sin^2\theta\,dr\,d\theta = \int(r')^2\left(\frac{3}{2}\cos^2\theta\right)(R-2r')\sin^2\theta\,dr\,d\theta - \frac{1}{2}\int(R-2r')(r')^2\sin^2\theta\,dr\,d\theta$$

$$\int_0^\pi \cos^2\theta\sin^2\theta\,d\theta = \frac{1}{4}\int_0^\pi(1+\cos 2\theta)(1-\cos 2\theta)\,d\theta = \frac{1}{4}\int_0^\pi(1-\cos^2 2\theta)\,d\theta$$

$$= \frac{\pi}{4}-\frac{1}{8}\int_0^\pi(1+\cos 4\theta)\,d\theta = \frac{\pi}{4}-\frac{\pi}{8} = \frac{\pi}{8}$$

$$\int(r')^2\left(\frac{3}{2}\cos^2\theta\right)(R-2r')\sin^2\theta\,dr\,d\theta = \frac{3}{2}\int(r')^2(R-2r')\,dr\int\cos^2\theta\sin^2\theta\,d\theta$$

$$\int(r')^2(R-2r')\,dr = \int\left(R(r')^2-2(r')^3\right)dr = \left[\frac{R}{3}(r')^3-\frac{(r')^4}{2}\right]_0^R = \frac{R^4}{3}-\frac{R^4}{2}$$

$$= \frac{2R^4-3R^4}{6} = -\frac{1}{6}R^4$$

$$\Rightarrow \int(r')^2\left(\frac{3}{2}\cos^2\theta-\frac{1}{2}\right)\rho(\vec{r}')\,d\tau' = 2\pi kR\left[\frac{3}{2}\left(-\frac{1}{6}R^4\right)\left(\frac{\pi}{8}\right)-\frac{1}{2}\left(-\frac{1}{6}R^4\right)\left(\frac{\pi}{2}\right)\right]$$

$$= 2\pi kR\left[\left(\frac{-R^4\pi}{32}\right)+\left(\frac{R^4\pi}{24}\right)\right] = \frac{\pi^2R^4k}{48}$$

So to 3rd order

$$V(\vec{r}) \simeq \frac{\pi R^4 k}{192\varepsilon_0 r^3}$$