3.20

$$\sigma(\theta) = \varepsilon_0 \left(\frac{\partial V_{in}}{\partial r} - \frac{\partial V_{out}}{\partial r} \right) \Big|_{r=R}$$

$$V_{in} = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta) \Rightarrow \frac{\partial V_{in}}{\partial r} = \sum_{l=1}^{\infty} l A_l r^{l-1} P_l(\cos\theta)$$

$$V_{out} = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta) \Rightarrow \frac{\partial V_{out}}{\partial r} = -\sum_{l=0}^{\infty} (l+1) \frac{B_l}{r^{l+2}} P_l(\cos\theta)$$

$$\frac{\partial V_{in}}{\partial r} - \frac{\partial V_{out}}{\partial r} = \frac{B_0 P_0(\cos\theta)}{r^2} + \sum_{l=1}^{\infty} \left[l A_l r^{l-1} + (l+1) \frac{B_l}{r^{l+2}} \right] P_l(\cos\theta)$$

$$A_l = \frac{2l+1}{2R^l} C_l \quad ; \quad B_l = \frac{2l+1}{2} R^{l+1} C_l$$

$$\frac{\sigma(\theta)}{\varepsilon_0} = \frac{B_0 P_0(\cos\theta)}{} + \left[\frac{2l^2+l}{2R^l} r^{l-1} + \frac{(2l+1)(l+1)R^{l+1}}{2r^{l+2}} \right] C_l P_l(\cos\theta)$$

$$(2l+1)(l+1) = 2l^2 + 2l + l + 1 = 2l^2 + 3l + 1$$

The bracketed expression is

$$\frac{(2l^2+l) r^{2l+1} + (2l^2+3l+1) R^{2l+1}}{2R^l (r^{l+2})} \Bigg|_{r=R} = (2l^2 + l + 2l^2 + 3l + 1) \frac{R^{2l+1}}{2R^{2l+2}}$$

$$= (2l+1)^2 \frac{1}{2R}$$

also,

$$\frac{B_0 P_0(\cos\theta)}{r^2} \Bigg|_{r=R} = \frac{2l+1}{2R^2} R^{l+1} C_l P_0 = \frac{1}{2R} C_0 P_0(\cos\theta) = \frac{(2l+1)^2}{2R} C_l P_l(\cos\theta) \Bigg|_{l=0}$$

So

$$\sigma(\theta) = \frac{\varepsilon_0}{2R} \sum_{l=0}^{\infty} (2l+1)^2 C_l P_l(\cos\theta)$$