Vinkle, Zachary | EM Griffiths | 07/14/18 | 1/1

3.7 Since we already know the potential for a single charge in such a configuration, we may apply the superposition principal to obtain the potential:

$$4\pi\varepsilon_0 V = \left[\frac{q}{\sqrt{x^2+y^2+(z-3d)^2}} - \frac{q}{\sqrt{x^2+y^2+(z+3d)^2}}\right] + \left[\frac{2q}{\sqrt{x^2+y^2+(z+d)^2}} - \frac{2q}{\sqrt{x^2+y^2+(z-d)^2}}\right]$$

and the force in the image problem is

$$F = -k\left[\frac{2q^2}{(2d)^2} - \frac{2q^2}{(4d)^2} + \frac{q^2}{(6d)^2}\right]\hat{z}$$

by intuition. Let's check that it matches $-q\nabla V$.

Set $k=1$ for convenience

$$-\nabla\left[\frac{q}{\sqrt{x^2+y^2+(z+d)^2}}\right] = q\,\frac{x\hat{x}+y\hat{y}+(z+d)\hat{z}}{\left(x^2+y^2+(z+d)^2\right)^{3/2}}$$

Hence

$$-\nabla V = q\left[\frac{x\hat{x}+y\hat{y}+(z-3d)\hat{z}}{(x^2+y^2+(z-3d)^2)^{3/2}} - \frac{x\hat{x}+y\hat{y}+(z+3d)\hat{z}}{(x^2+y^2+(z+3d))^{3/2}}\right]$$
$$+2q\left[\frac{x\hat{x}+y\hat{y}+(z+d)\hat{z}}{(x^2+y^2+(z+d)^2)^{3/2}} - \frac{x\hat{x}+y\hat{y}+(z-d)\hat{z}}{(x^2+y^2+(z-d)^2)^{3/2}}\right]$$

So for $+q$, $x=y=0$, $z=3d$

then

$$E(0,0,3d) = -\nabla V(0,0,3d) = q\left[0 - \frac{6d\hat{z}}{[(d6)^2]^{3/2}}\right] + 2q\left[\frac{4d\hat{z}}{[(4d)^2]^{3/2}} - \frac{2d\hat{z}}{[(2d)^2]^{3/2}}\right]$$

$$= \left[\frac{-q}{(6d)^2} + \frac{2q}{(4d)^2} - \frac{2q}{(2d)^2}\right]\hat{z}$$

so $F = q\left[\frac{-q}{(6d)^2} + \frac{2q}{(4d)^2} - \frac{2q}{(2d)^2}\right]\hat{z} = -\left[\frac{2q^2}{(2d)^2} - \frac{2q^2}{(4d)^2} + \frac{q}{(6d)^2}\right]\hat{z}$

reintroducing k,

$$F = -k\left[\frac{2q^2}{(2d)^2} - \frac{2q^2}{(4d)^2} + \frac{q}{(6d)^2}\right]\hat{z}$$

which is the same as above □.