Hinkle, Zachary | EM Griffiths | 07/09/18 | 1/1

2.38

a) The surface charges are

$$\sigma_R = \frac{q}{4\pi R^2} \; ; \quad \sigma_a = \frac{-q}{4\pi a^2} \; ; \quad \sigma_b = \frac{q}{4\pi b^2}$$

b)
$$E = \begin{cases} \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} & \text{for } r \geq b,\ R \leq r \leq a \\ 0 & \text{for } a < r < b,\ r < R \end{cases}$$

let $k = \frac{1}{4\pi\varepsilon_0}$

$$V = \int_0^\infty \vec{E}\cdot\vec{dl} = \int_0^R \vec{E}\cdot\vec{dl} + \int_R^a \vec{E}\cdot\vec{dl} + \int_a^b \vec{E}\cdot\vec{dl} + \int_b^\infty \vec{E}\cdot\vec{dl} = -kq\left[\int_a^R r^{-2}dr + \int_\infty^b r^{-2}dr\right]$$

$$V = kq\left[\left(\frac{1}{r}\right)_a^R + \left(\frac{1}{R}\right)_\infty^b\right] = kq\left[\frac{1}{R} - \frac{1}{a} + \frac{1}{b}\right] = kq\left[\frac{ab - bR + aR}{abR}\right]$$

$$V = q\,\frac{ab - bR + aR}{4\pi\varepsilon_0 abR}$$

c) The question is unclear but I assume by "outer surface" we mean the outer surface of the shell.

$$V(\infty) = V(b) \Rightarrow \int_b^\infty \vec{E}\cdot\vec{dl} = 0 \Rightarrow V = \int_R^a \vec{E}\cdot\vec{dl} = kq\left[\frac{1}{R} - \frac{1}{a}\right]$$

$$\Rightarrow V = q\,\frac{a - R}{4\pi\varepsilon_0 aR}$$

and σ_R and σ_a remain the same, but σ_b becomes zero.