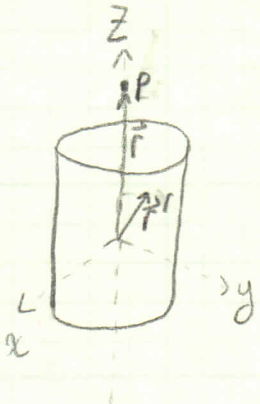
2.27



$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\,d\tau'$$

$\rho = \text{const.}$

$d\vec{\tau} = s\,d\theta\,ds\,dz$

$\vec{r}\,' = s\hat{s} + z\hat{z} = s\cos\theta\hat{x} + s\sin\theta\hat{y} + z\hat{z}$

$\vec{r} = p\hat{z}$

$\vec{\mathscr{r}} = r - \vec{r}\,' = -s\hat{s} + (p-z)\hat{z}$

$\mathscr{r}^2 = s^2 + p^2 - 2pz + z^2$

$\mathscr{r} = \sqrt{s^2 + p^2 - 2pz + z^2}$

$$V(\vec{r}) = \frac{\rho}{4\pi\varepsilon_0}\int \frac{s\,d\theta\,ds\,dz}{\sqrt{s^2+p^2-2pz+z^2}} = \frac{\rho}{2\varepsilon_0}\int \frac{s\,ds\,dz}{\sqrt{s^2+p^2-2pz+z^2}}$$

let $u = s^2 + p^2 - 2pz + z^2$ $\quad du = 2s\,ds$

$$V(\vec{r}) = \frac{\rho}{4\varepsilon_0}\int u^{-1/2}\,du\,dz = \frac{\rho}{2\varepsilon_0}\int_{-L/2}^{L/2} dz\left(\sqrt{R^2+p^2-2pz+z^2} - \sqrt{p^2-2pz+z^2}\right)$$

$$\int\sqrt{ax^2+bx+c}\,dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right|$$

here $a = 1$; $b = -2p$; $c = p^2$ or $p^2 + R^2$

$$\left[\frac{2z-2p}{4}\sqrt{z^2-2pz+p^2+R^2} + \frac{R^2}{2}\ln\left|2z-2p+2\sqrt{z^2-2pz+R^2+p^2}\right| - \frac{2z-2p}{2}\sqrt{z^2-2pz+p^2}\right]_{-\ell}^{\ell}$$

where $\ell = L/2$

2.27 $$\frac{2\varepsilon_0}{\rho} = V(\vec{r}) = \frac{l-P}{2}\sqrt{l^2-2pl+p^2+R^2} + \frac{R^2}{2}\ln\left|2l-2p+2\sqrt{l^2+2pl+R^2+p^2}\right| - \frac{l-P}{2}\sqrt{l^2-2pl+p^2} + \frac{l+P}{2}\sqrt{l^2+2pl+P^2+R^2} - \frac{R^2}{2}\ln\left|-2l-2p+2\sqrt{l^2+2pl+R^2+P^2}\right| + \frac{l+P}{2}\sqrt{l^2+2pl+P^2}$$

$$\frac{2\varepsilon_0}{\rho}V = \frac{l-P}{2}\left[\sqrt{l^2-2pl+P^2+R^2} - \sqrt{l^2-2pl+P^2}\right] + \frac{l+P}{2}\left[\sqrt{l^2-2pl+p^2+R^2} - \sqrt{l^2-2pl+P^2}\right] + \frac{R^2}{2}\ln\left|-\frac{l-P+\sqrt{l^2+2pl+R^2+P^2}}{l+P-\sqrt{l^2+2pl+R^2+P^2}}\right|$$

Let $a = l-P$
$b = l+P$
$c = l^2+2pl+R^2+P^2$

note
$ab = (l-P)(l+P) = l^2-P^2$
$b^2-c = -R^2$
$a+b = 2l$

$$\frac{2\varepsilon_0}{\rho}V(\vec{r}) = \left(\frac{l-P+l+P}{2}\right)\left[\sqrt{c} - \sqrt{c-R^2}\right] + \frac{R^2}{2}\ln\left|-\frac{(a+\sqrt{c})(b+\sqrt{c})}{(b-\sqrt{c})(b+\sqrt{c})}\right|$$

$$= l\left[\sqrt{c}-\sqrt{c-R^2}\right] + \frac{R^2}{2}\ln\left|-\frac{ab+a\sqrt{c}+b\sqrt{c}+c}{b^2-c}\right|$$

$$\frac{2\varepsilon_0}{\rho}V(\vec{r}) = l\left[\sqrt{l^2+2pl+R^2+P^2} - \sqrt{l^2+2pl+P^2}\right] + \frac{R^2}{2}\ln\left|\frac{l^2-P^2+2l\sqrt{l^2+2pl+R^2+P^2}+l^2+2pl+R^2+P^2}{R^2}\right|$$

$$\frac{2\varepsilon_0}{\rho}V(\vec{r}) = l\left(\sqrt{l^2+2Pl+R^2+P^2} - \sqrt{l^2+2Pl+P^2}\right) +$$

$$+\frac{R^2}{2}\ln\left|\frac{2l^2+2Pl+R^2+2l\sqrt{l^2+2Pl+R^2+P^2}}{R^2}\right|$$

$$\vec{E} = \frac{\partial V(\vec{r})}{\partial z}\hat{z} \text{ ; here, } z = P \text{ so}$$

$$\vec{E} = \frac{\partial V(\vec{r})}{\partial P}\hat{z}$$

$$\hat{z}\cdot\frac{2\varepsilon_0}{\rho}\frac{\partial V(\vec{r})}{\partial P} = l\left(\frac{2P+2l}{\sqrt{C}} - \frac{2P+2l}{\sqrt{C-R^2}}\right) + \frac{R^2\left(2l+2l\frac{2P+2l}{\sqrt{C}}\right)}{2\left(\frac{2l^2+2Pl+R^2+2l\sqrt{C}}{R^2}\right)}$$

$$\hat{z}\cdot\frac{2\varepsilon_0}{\rho}\frac{\partial V(\vec{r})}{\partial P} = l\left(\frac{(2P+2l)\sqrt{C-R^2}}{\sqrt{C^2-CR^2}} - \frac{(2P+2l)\sqrt{C}}{\sqrt{C^2-CR^2}}\right) + \frac{R^4}{2}\cdot\frac{\frac{2l\sqrt{C}+4lP+4l^2}{\sqrt{C}}}{2l^2+2Pl+R^2+2l\sqrt{C}}$$

$$\hat{z}\cdot\frac{2\varepsilon_0}{\rho}\frac{\partial V(\vec{r})}{\partial P} = 2l(P+l)\left(\frac{\sqrt{C-R^2}-\sqrt{C}}{\sqrt{C^2-CR^2}}\right) + \frac{R^4(2l\sqrt{C}+4lP+4l^2)}{2(2l^2\sqrt{C}+2Pl\sqrt{C}+R^2\sqrt{C}+2lC)}$$