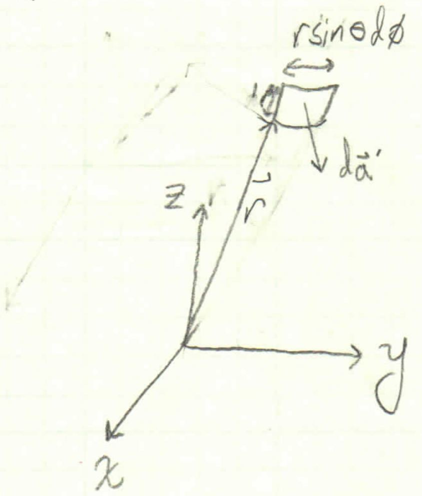
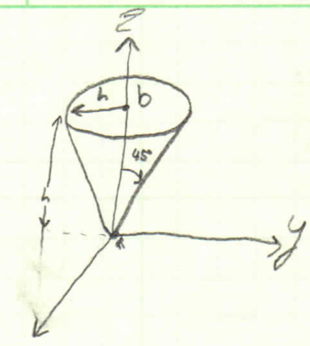
2.26 $V = \frac{\sigma}{4\pi\varepsilon_0}\int_a^b \frac{da'}{\mathscr{r}}$; $\vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$;





$d\vec{a}\,' = r\sin\theta\, dr\, d\phi\, \hat{\theta}$

$da' = r\sin\theta\, dr\, d\phi = \frac{\sqrt{2}\, r\, dr\, d\theta}{2}$

$\vec{r} = z\hat{z}$; $r' = r\hat{r} = r(\sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z})$

$\vec{\mathscr{r}} = -r\sin\theta\cos\phi\,\hat{x} - r\sin\theta\sin\phi\,\hat{y} + (z - r\cos\theta)\hat{z}$

$|\mathscr{r}|^2 = r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + z^2 - 2zr\cos\theta + r^2\cos^2\theta$

$= r^2 + z^2 - 2zr\cos\theta \overset{\theta = 45^\circ}{\Rightarrow} r^2 + z^2 - \sqrt{2}zr$.

$|\mathscr{r}| = \sqrt{r^2 + z^2 - \sqrt{2}zr}$

$V(z) = \frac{\sigma}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{\sqrt{2}h} \frac{r\sin\theta\, dr\, d\phi}{\sqrt{r^2 + z^2 - \sqrt{2}zr}} = \frac{\sqrt{2}\sigma}{4\varepsilon_0}\int_0^{\sqrt{2}h} \frac{r\, dr}{\sqrt{r^2 + z^2 - \sqrt{2}zr}}$

$\int \frac{x\, dx}{\sqrt{ax^2 + bx + c}} = \frac{1}{a}\sqrt{ax^2 + bx + c} - \frac{b}{2a^{3/2}}\ln\left|2ax + b + 2\sqrt{a(ax^2 + bx + c)}\right|$

$\left[\sqrt{r^2 - \sqrt{2}zr + z^2} + \frac{\sqrt{2}z}{2}\ln\left|2r - \sqrt{2}z + 2\sqrt{r^2 - \sqrt{2}zr + z^2}\right|\right]_0^{\sqrt{2}h}$

$\sqrt{2h^2 - 2hz + z^2} - z$

$\frac{z}{\sqrt{2}}\ln\left|\frac{2\sqrt{2}h - \sqrt{2}z + 2\sqrt{2h^2 - 2hz + z^2}}{2z - \sqrt{2}z}\right|$

2.26 so,

$$\frac{4\varepsilon_0}{\sqrt{2}\sigma} V(z) = \sqrt{2h^2 - 2hz + z^2} - z + \frac{z}{\sqrt{2}} \ln\left| \frac{2\sqrt{2}h - \sqrt{2}z + 2\sqrt{2h^2 - \sqrt{2}hz + z^2}}{z(2-\sqrt{2})} \right|$$

then

$$\frac{4\varepsilon_0}{\sqrt{2}\sigma} V(0) = \sqrt{2}h$$

$$\frac{4\varepsilon_0}{\sqrt{2}\sigma} V(h) = h - h + \frac{h}{\sqrt{2}} \ln\left| \frac{h2\sqrt{2} - h\sqrt{2} + 2h}{h(2-\sqrt{2})} \right|$$

$$= \frac{h}{\sqrt{2}} \ln\left| \frac{h(\sqrt{2}+2)}{h(2-\sqrt{2})} \right| = \frac{h}{\sqrt{2}} \ln\left| \frac{(2+\sqrt{2})^2}{4-2} \right| = \frac{h}{\sqrt{2}} \ln\left| \frac{1}{2}(2+\sqrt{2})^2 \right|$$

$$= \frac{h}{\sqrt{2}} \ln\left[(\sqrt{2}+1)^2\right] = \frac{2h}{\sqrt{2}} \ln(1+\sqrt{2}) = \sqrt{2}h \ln(1+\sqrt{2})$$

So $\frac{4\varepsilon_0}{\sqrt{2}\sigma}\left[V(h) - V(0)\right] = \sqrt{2}h\left(\ln(1+\sqrt{2}) - 1\right)$

So

$$V(h) - V(0) = \frac{2h\sigma}{4\varepsilon_0}\left[\ln(\sqrt{2}+1) - 1\right]$$

$$V(h) - V(0) = \frac{h\sigma}{2\varepsilon_0}\left[\ln(\sqrt{2}+1) - 1\right]$$