2.24 & 2.25

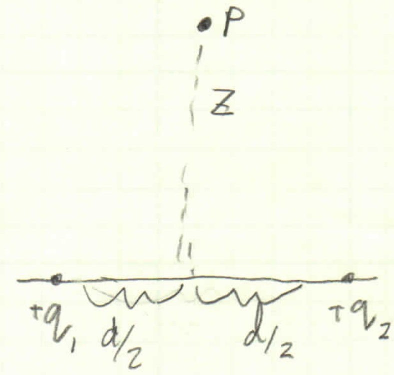
From problem 2.16, $E = \begin{cases} \frac{\rho}{2\varepsilon_0} s\hat{s} & \text{for } 0 \le s \le a \\ \frac{\rho a^2}{2\varepsilon_0}\frac{\hat{s}}{s} & \text{for } a < s < b \\ 0 & \text{for } b < s \end{cases}$

$$V(b)-V(0) = \int_0^b \vec{E}\cdot d\vec{l} = -\frac{\rho}{2\varepsilon_0}\int_0^a s\,ds - \frac{\rho a^2}{2\varepsilon_0}\int_a^b \frac{ds}{s} = -\frac{\rho}{2\varepsilon_0}\frac{1}{2}s^2\Big|_0^a - \frac{\rho a^2}{2\varepsilon_0}\ln(s)\Big|_a^b$$

$$= -\left(\frac{\rho}{4\varepsilon_0}a^2 + \frac{\rho a^2}{2\varepsilon_0}\left[\ln(b)-\ln(a)\right]\right) = -\frac{\rho a^2}{4\varepsilon_0}\left(2\ln\left(\frac{b}{a}\right)+1\right)$$

2.25

a) $V(r) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$



$\vec{r}_1 = z\hat{z} + \frac{d}{2}\hat{x}$

$r_1 = \sqrt{z^2 + \frac{d^2}{4}} = \frac{1}{2}\sqrt{4z^2+d^2}$

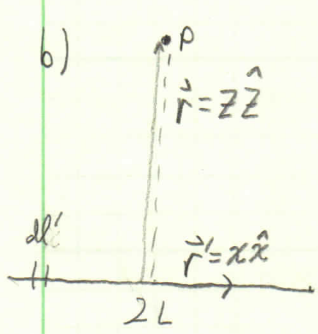
$\vec{r}_2 = z\hat{z} - \frac{d}{2}\hat{x}$

$r_2 = \sqrt{z^2+\frac{d^2}{4}} = \frac{1}{2}\sqrt{4z^2+d^2}$

so $q_1 = q_2$ & $r_1 = r_2$

$V(z\hat{z}) = \frac{1}{4\pi\varepsilon_0}\frac{2q}{\frac{1}{2}\sqrt{4z^2+d^2}}$; $-\nabla V = \frac{1}{4\pi\varepsilon_0}\frac{16qz}{(4z^2+d^2)^{3/2}}\hat{z}$

b)



$V = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda(r')}{r}dl'$

$\vec{r} = \vec{r} - \vec{r}' = z\hat{z} - x\hat{x}$; $r = \sqrt{z^2+x^2}$; $dl' = dx$

$\lambda = \text{const.}$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L}^{L}\frac{dx}{\sqrt{z^2+x^2}} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left|x+\sqrt{z^2+x^2}\right|\Big|_{-L}^{L} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left|\frac{L+\sqrt{z^2+L^2}}{-L+\sqrt{z^2+L^2}}\right|$$

$$\Rightarrow \frac{4\pi\varepsilon_0}{\lambda}V = \ln\left|L+\sqrt{z^2+L^2}\right| - \ln\left|-L+\sqrt{z^2+L^2}\right|$$

2.25 b)

$$\frac{4\pi\epsilon_0}{\lambda}\frac{\partial V}{\partial z} = \frac{z}{(L+\sqrt{z^2+L^2})\sqrt{z^2+L^2}} - \frac{z}{(-L+\sqrt{z^2+L^2})\sqrt{z^2+L^2}}$$

let $u = \sqrt{z^2+L^2}$

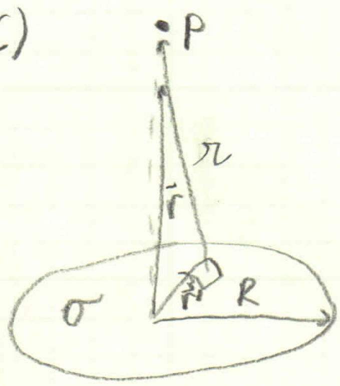
$$= \frac{z}{Lu+u^2} + \frac{z}{Lu-u^2} = \frac{2zLu}{(Lu+u^2)(Lu-u^2)}$$

$$= \frac{2zLu}{L^2u^2-u^4} = \frac{2zLu}{L^4+L^2z^2-L^4+2L^2z^2-z^4} = \frac{-2zLu}{z^4+L^2z^2}$$

$$= \frac{-2zLu}{z^2(L^2+z^2)} = -\frac{2Lu}{zu^2} = -\frac{2L}{zu}$$

$$\Rightarrow E = -\nabla V = \frac{\lambda}{4\pi\epsilon_0}\frac{2L}{z\sqrt{z^2+L^2}}\hat{z}$$

c)



$r' = s\hat{s} \qquad \vec{\mathscr{r}} = z\hat{z} - s\hat{s}$

$r = z\hat{z} \qquad \mathscr{r} = \sqrt{z^2+s^2}$

$da' = s\,d\theta\,ds$

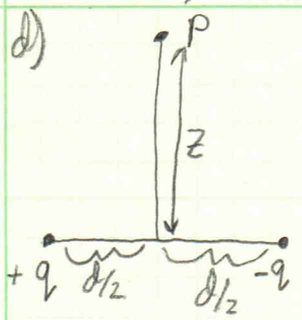
$$V = \frac{\sigma}{4\pi\epsilon_0}\int_0^R\int_0^{2\pi}\frac{s\,d\theta\,ds}{\sqrt{z^2+s^2}} = \frac{\sigma}{2\epsilon_0}\int_0^R\frac{s\,ds}{(z^2+s^2)^{1/2}} = \frac{\sigma}{2\epsilon_0}\sqrt{s^2+z^2}\Big|_0^R$$

$$V = \frac{\sigma}{2\epsilon_0}\left[\sqrt{R^2+z^2} - z\right]$$

$$\frac{2\epsilon_0}{\sigma}\frac{\partial V}{\partial z} = \frac{z}{\sqrt{R^2+z^2}} - 1 = \frac{z-\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}}$$

$$\Rightarrow E = -\nabla V = \frac{\sigma}{2\epsilon_0}\left[\frac{\sqrt{R^2+z^2}-z}{\sqrt{R^2+z^2}}\right]\hat{z}$$

2.25 d)



$$\mathscr{r}_1 = \mathscr{r}_2 = \frac{1}{2}\sqrt{4z^2+d^2}$$

$$V(z\hat{z}) = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{q_1}{\mathscr{r}_1} + \frac{q_2}{\mathscr{r}_2}\right] = \frac{\lambda}{4\pi\epsilon_0}\frac{1}{\mathscr{r}_1}\left[q-q\right] = 0$$

Suggests $\vec{E} = 0$ at point P