2.4 Suppose instead $\vec{\mathscr{r}} = -x\hat{x} - y\hat{y} + z\hat{z}$ $\quad \mathscr{r} = \sqrt{x^2+y^2+z^2}$

in Example 2, the point P is also a distance y behind the wire

$$\hat{\mathscr{r}} = \frac{-x\hat{x} - y\hat{y} + z\hat{z}}{\sqrt{x^2+y^2+z^2}}$$

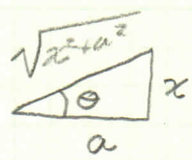
$$E = k\int \frac{\lambda(x)}{\mathscr{r}^2}\hat{\mathscr{r}}\, dl' \; ; \; \lambda(x) = \text{const}, \quad dl' = dx$$

$$E = k\lambda \int_{-L}^{L} \frac{-x\hat{x} - y\hat{y} + z\hat{z}}{(x^2+y^2+z^2)^{3/2}}\, dx$$

let $u = x^2+y^2+z^2 \Rightarrow du = 2x\,dx$

$$\Rightarrow -\int \frac{x\hat{x}}{(x^2+y^2+z^2)^{3/2}} dx = -\frac{\hat{x}}{2}\int_{L^2+y^2+z^2}^{L^2+y^2+z^2} u^{-3/2}\, du = 0$$

let $a^2 = y^2+z^2$; $\quad x = a\tan(\theta)$, $\quad dx = a\sec^2(\theta)\,d\theta$



$$-\int_{-L}^{L} \frac{y\hat{y}}{(x^2+y^2+z^2)^{3/2}} dx = -y\int \frac{a\sec^2(\theta)\,d\theta}{(a^2+a^2\tan^2(\theta))^{3/2}}\hat{y} = -y\int \frac{a\sec^2(\theta)\,d\theta}{a^3\sec^3(\theta)}\hat{y}$$

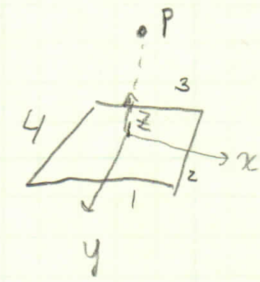
$$= -\frac{y}{a^2}\int \cos\theta\, d\theta = -\frac{y}{a^2}\sin\theta = \left.\frac{-yx}{a^2\sqrt{x^2+a^2}}\right|_{-L}^{L} = \frac{-2Ly}{a^2\sqrt{L^2+y^2+z^2}}$$

Similarly for Z w/ sub $-y \to z$ in numerator

$$\int_{-L}^{L} \frac{z\hat{z}}{(x^2+y^2+z^2)^{3/2}} dx = \frac{2Lz}{a^2\sqrt{L^2+y^2+z^2}}$$

so $$E = k\lambda\frac{2L(z-y)}{(y^2+z^2)\sqrt{L^2+y^2+z^2}}$$

2.4 Applying this eq. in y direction, noting that $L = \frac{a}{2}$



$$E_1 = k\lambda \frac{a\left(z - \frac{a}{2}\right)}{\left(\frac{a^2}{4}+z^2\right)\sqrt{\frac{a^2}{4}+\frac{a^2}{4}+z^2}}$$

$$E_3 = k\lambda \frac{a\left(z + \frac{a}{2}\right)}{\left(\frac{a^2}{4}+z^2\right)\sqrt{\frac{a^2}{4}+\frac{a^2}{4}+z^2}}$$

$E_2 = E_1$; $E_4 = E_3$ by symmetry

So

$$E_p = 2(E_1+E_2) = 2k\lambda \frac{2az^2}{\left(\frac{a^2}{4}+z^2\right)\frac{1}{2}\sqrt{2a^2+4z^2}} = k\lambda \frac{32az}{(a^2+4z^2)\sqrt{2a^2+4z^2}}$$