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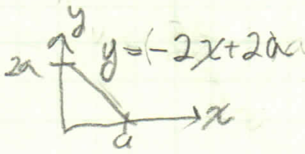
1.58 $\vec{V} = y\hat{z}$

$$\vec{\nabla}\times\vec{V} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & y \end{vmatrix} = y \begin{vmatrix} \hat{x} & \hat{y} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{vmatrix} = \hat{x}$$

the eq. of the plane is $2x + y + 2z = 2a \Rightarrow z = a - x - \frac{1}{2}y$

$d\vec{a} = \vec{n}\,ds$; $ds = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\; da$; $\vec{n} = \pm \left\langle \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1 \right\rangle$

$d\vec{a} = \vec{n}\,da = -\langle -1, -\frac{1}{2}, -1 \rangle\, dy\,dx = (\hat{x} + \frac{1}{2}\hat{y} + \hat{z})\, dy\,dx$



$(\vec{\nabla}\times\vec{V})\cdot d\vec{a} = dy\,dx$

$$\int_0^a \int_0^{-2x+2a} dy\,dx = \int_0^a -2x + 2a = \left[-x^2 + 2ax\right]_0^a = -a^2 + 2a^2 = a^2$$

Line Integrals

First path (xy-plane)

$y = -2x + 2a$; $z = 0 \Rightarrow dy = -2dx \Rightarrow d\vec{l} = dx\,\hat{x} - 2dx\,\hat{y}$

$\Rightarrow \vec{V}\cdot d\vec{l} = 0$

2nd Path (yz-plane)

$x = 0$; $z = -\frac{1}{2}y + a \Rightarrow dz = -\frac{1}{2}dy \Rightarrow d\vec{l} = dy\,\hat{y} - \frac{1}{2}dy\,\hat{z}$

$\vec{V}\cdot d\vec{l} = -\frac{1}{2}y\,dy$

$-\frac{1}{2}\int_{2a}^{0} y\,dy = -\frac{1}{4}y^2\Big|_{2a}^{0} = a^2$

3rd Path (xz-plane)

$y = 0 \therefore \vec{V}\cdot d\vec{l} = 0$

$\Rightarrow \oint_P \vec{V}\cdot d\vec{l} = a^2$