1.55 $\vec{V} = ay\hat{x} + bx\hat{y}$

$$\int_S (\vec{\nabla}\times\vec{V})\cdot d\vec{a} = \oint_P \vec{V}\cdot d\vec{l}$$

$$\vec{\nabla}\times\vec{V} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ay & bx & 0 \end{vmatrix} = \hat{\imath}(0-0) + \hat{\jmath}(0-0) + \hat{k}(b-a)$$

$$d\vec{a} = r\,dr\,d\theta$$

$$\int_0^R\int_0^{2\pi} r(b-a)\,dr\,d\theta = 2\pi\left(\tfrac{1}{2}R^2\right) = \pi R^2(b-a)$$

$$d\vec{l} = R\,d\phi\,\hat{\phi} = R\,d\phi(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y})$$

$$\oint_0^{2\pi} (ay\hat{x} + bx\hat{y})\cdot(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y})\,R\,d\phi = \oint_0^{2\pi}(-ay\sin\phi + bx\cos\phi)\,R\,d\phi$$

$$x = R\cos\phi \ ; \ y = R\sin\phi$$

$$\oint_0^{2\pi}(-Ra\sin^2\phi + bR\cos^2\phi)\,r\,d\phi = R^2\int_0^{2\pi}\left[b\cos^2\phi - a\sin^2\phi\right]d\phi$$

$$= \frac{R^2}{2}\int_0^{2\pi}\left[b(1+\cos 2\phi) - a(1-\cos 2\phi)\right]d\phi = \frac{R^2}{2}\left[b\left(\phi + \tfrac{1}{2}\sin 2\phi\right) - a\left(\phi - \tfrac{1}{2}\sin 2\phi\right)\right]_0^{2\pi}$$

$$= \frac{R^2}{2}\left[2\pi b - 2\pi a\right] = \pi R^2(b-a)$$