Table of Integrals Common in QM

1 Gaussian Integrals

$$\int_0^\infty xe^{-x/a}dx = a^2 \tag{1}$$

$$\int_0^\infty x^n e^{-x/a}dx = n!a^{n+1} \tag{2}$$

$$\int_0^\infty e^{-x^2/a^2}dx = \sqrt{\pi}(\frac{a}{2}) \tag{3}$$

$$\int_0^\infty xe^{-x^2/a^2}dx = \frac{a^2}{2} \tag{4}$$

$$\int_0^\infty x^2 e^{-x^2/a^2}dx = \sqrt{\pi}(\frac{a^3}{4}) \tag{5}$$

$$\int_0^\infty x^{2n} e^{-x^2/a^2}dx = \sqrt{\pi}\frac{(2n)!}{n!}(\frac{a}{2})^{2n+1} \tag{6}$$

$$\int_0^\infty x^{2n+1} e^{-x^2/a^2}dx = \frac{n!}{2}a^{2n+2} \tag{7}$$

2 Gaussian Integrals

$$\int x\sin axdx = \frac{ax}{a^2} - \frac{x\cos ax}{a} \tag{8}$$

$$\int x^2\sin axdx = \frac{2-a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a} \tag{9}$$

$$\int \sin^2 axdx = \frac{x}{2} - \frac{\sin 2ax}{4a} \tag{10}$$

$$\int x\sin^2 axdx = \frac{x^2}{4} - \frac{x\sin 2ax}{4a} + \frac{\sin^2 ax}{4a^2} \tag{11}$$

$$\int x\cos axdx = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a} \tag{12}$$

$$\int x^2\cos axdx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2-2}{a^3}\sin ax \tag{13}$$

$$\int \cos^2 axdx = \frac{x}{2} + \frac{\sin 2ax}{4a} \tag{14}$$

$$\int x\cos^2 axdx = \frac{x^2}{4} + \frac{x\sin 2ax}{4a} - \frac{sin^2 ax}{4a^2} \tag{15}$$

The Trig Identities I don't have Memorized

Even/Odd Relations

$$\sin(-\theta) = -\sin(\theta) \rightarrow -\sin(-\theta) = \sin(\theta) \tag{16}$$

$$\cos(-\theta) = \cos(\theta) \rightarrow -\cos(-\theta) = -\cos(\theta) \tag{17}$$

Product Identities

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)] \tag{18}$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)] \tag{19}$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha+\beta) - \cos(\alpha-\beta)] \tag{20}$$

Power Reduction Formulae

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \tag{21}$$

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta) \tag{22}$$

$$\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta \tag{23}$$

$$\cos^3\theta = \frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta \tag{24}$$

Sum and Difference Formulae

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \tag{25}$$

$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \pm \sin\alpha\sin\beta \tag{26}$$

Double and Triple Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta \tag{27}$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^{\theta}\end{aligned} \tag{28}$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta \tag{29}$$

$$\cos 3\theta = -3\cos\theta + 4\cos^3\theta \tag{30}$$