1.15 & 1.16 | Hinkle, Zachary | QM Sakurai | 08/29/18 | 1/1

1.15 Since $|a', b'\rangle$ forms a complete basis, we know that

$$1 = \sum_{a'} \sum_{b'} |a', b'\rangle\langle a', b'|$$

So,

$$AB = \sum_{a''} \sum_{b''} \sum_{a'} \sum_{b'} |a'', b''\rangle\langle a'', b''|AB|a', b'\rangle\langle a', b'|$$

$$= \sum_{a''} \sum_{b''} \sum_{a'} \sum_{b'} |a'', b''\rangle\langle a'', b''|Ab'|a', b'\rangle\langle a', b'|$$

$$= \sum_{a''} \sum_{b''} \sum_{a'} \sum_{b'} |a'', b''\rangle\langle a'', b''|b'a'|a', b'\rangle\langle a', b'|$$

$$= \sum_{a'} \sum_{b'} a'b' |a', b'\rangle\langle a', b'|$$

Similarly

$$BA = \sum_{a''} \sum_{b''} \sum_{a'} \sum_{b'} |a'' b''\rangle\langle a'' b''|BA|a', b'\rangle\langle a', b'|$$

$$= \sum_{a''} \sum_{b''} \sum_{a'} \sum_{b'} |a'', b''\rangle\langle a'', b''|Ba'|a', b'\rangle\langle a', b'|$$

$$= \sum_{a''} \sum_{b''} \sum_{a'} \sum_{b'} |a'', b''\rangle\langle a'', b''|a'b'|a', b'\rangle\langle a', b'|$$

$$= \sum_{a'} \sum_{b'} a'b' |a', b'\rangle\langle a', b'|$$

So clearly $[A,B] = AB - BA = 0$

1.16 No. Given that $\{A, B\} = AB + BA = 0 \Rightarrow BA = -AB$

so

$[A, B] = AB - BA = AB + AB = 2AB$. For A & B to have simultaneously observables, it is necessarily that $[A, B] = 0$. Therefore A & B can only have simultaneous observables in the trivial case $AB = 0$