

$$1.12 \quad |\hat{n}; +\rangle = \cos \frac{\beta}{2} |+\rangle + e^{i\alpha} \sin \frac{\beta}{2} |-\rangle$$

and for this problem $\alpha=0$, $\beta=\gamma$

so

$$|\hat{n}; +\rangle = \cos \frac{\gamma}{2} |+\rangle + \sin \frac{\gamma}{2} |-\rangle$$

$$\langle S_x; + | \hat{n}; + \rangle = \cos \frac{\gamma}{2} \langle S_x; + | + \rangle + \sin \frac{\gamma}{2} \langle S_x; + | - \rangle$$

$$\langle S_x; + | \pm \rangle = \frac{1}{\sqrt{2}}$$

so

$$\langle S_x; + | \hat{n}; + \rangle = \frac{1}{\sqrt{2}} (\cos \frac{\gamma}{2} + \sin \frac{\gamma}{2})$$

$$|\langle S_x; + | \hat{n}; + \rangle|^2 = \frac{1}{2} (\cos \frac{\gamma}{2} + \sin \frac{\gamma}{2})^2 = \frac{1}{2} (1 + 2 \cos \frac{\gamma}{2} \sin \frac{\gamma}{2}) = \frac{1}{2} (1 + \sin \gamma)$$

$$(\Delta S_x)^2 = \langle S_x^2 \rangle - \langle S_x \rangle^2$$

$$\langle S_x^2 \rangle = \langle \hat{n}; + | S_x^2 | \hat{n}; + \rangle$$

$$S_x = \frac{\hbar}{2} (|+\rangle \langle -| + |-\rangle \langle +|)$$

$$S_x^2 = \frac{1}{2} \{ S_x | S_x \} = \left(\frac{\hbar}{2} \right)^2 \mathbb{1}$$

so

$$\langle S_x^2 \rangle = \left(\frac{\hbar}{2} \right)^2 (|\langle \hat{n}; + | + \rangle|^2 + |\langle \hat{n}; + | - \rangle|^2)$$

$$= \left(\frac{\hbar}{2} \right)^2 (\cos^2 \frac{\gamma}{2} + \sin^2 \frac{\gamma}{2}) = \left(\frac{\hbar}{2} \right)^2$$

$$\langle S_x \rangle = \frac{\hbar}{2} (\langle \hat{n}; + | + \rangle \langle - | \hat{n}; + \rangle + \langle \hat{n}; + | - \rangle \langle + | \hat{n}; + \rangle)$$

$$= \frac{\hbar}{2} (\cos \frac{\gamma}{2} \sin \frac{\gamma}{2} + \sin \frac{\gamma}{2} \cos \frac{\gamma}{2}) = \frac{\hbar}{2} \sin \gamma$$

$$\Rightarrow \langle S_x \rangle^2 = \left(\frac{\hbar}{2} \right)^2 \sin^2 \gamma \quad \text{so} \quad (\Delta S_x)^2 = \left(\frac{\hbar}{2} \right)^2 (1 - \sin^2 \gamma)$$