1.9 $\vec{S} = S_x \hat{\imath} + S_y \hat{\jmath} + S_z \hat{k}$, $\hat{n} = \sin\beta \cos\alpha\, \hat{\imath} + \sin\beta \sin\alpha\, \hat{\jmath} + \cos\beta\, \hat{k}$

Then $\langle \pm | \vec{S}\cdot\vec{n} | \pm \rangle = n_x \langle \pm | S_x | \pm \rangle + n_y \langle \pm | S_y | \pm \rangle + n_z \langle \pm | S_z | \pm \rangle$

So, $\langle \pm | \vec{S}\cdot\vec{n} | \pm \rangle = \hat{n}\cdot\vec{\sigma}$

$$\sigma_x = \langle \pm | S_x | \pm \rangle \doteq \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_y = \frac{i\hbar}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_z = \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

So

$$\langle \pm | \vec{S}\cdot\hat{n} | \pm \rangle = \frac{\hbar}{2}\begin{bmatrix} n_z & n_x - i n_y \\ n_x + i n_y & -n_z \end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix} \cos\beta & \sin\beta\, e^{-i\alpha} \\ \sin\beta\, e^{i\alpha} & -\cos\beta \end{bmatrix}$$

$$-(n_z - \lambda)(n_z + \lambda) - (n_x - i n_y)(n_x + i n_y) = 0$$

$$(n_z^2 + \lambda^2) + (n_x^2 + n_y^2) = 0$$

$$-\lambda^2 + n_z^2 + (n_y^2 + n_x^2) = 0$$

$$n_x^2 + n_y^2 = \sin^2\beta \cos^2\alpha + \sin^2\beta \sin^2\alpha = \sin^2\beta$$

$$\Rightarrow -\lambda^2 + n_z^2 + (n_x^2 + n_y^2) = -\lambda^2 + \cos^2\beta + \sin^2\beta$$

$$\Rightarrow \lambda = \pm 1$$

$$\begin{bmatrix} \cos\beta - 1 & \sin\beta\, e^{-i\alpha} \\ \sin\beta\, e^{i\alpha} & -\cos\beta - 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 0$$

$$a(\cos\beta - 1) + b \sin\beta\, e^{-i\alpha} = 0$$

$$b \sin\beta\, e^{-i\alpha} = a(1 - \cos\beta)$$

$$b = a e^{i\alpha}\left(\frac{1 - \cos\beta}{\sin\beta}\right)$$

$$\frac{1-\cos\beta}{\sin\beta} = \frac{2\sin^2\frac{\beta}{2}}{2\sin\frac{\beta}{2}\cos\frac{\beta}{2}} = \frac{\sin\frac{\beta}{2}}{\cos\frac{\beta}{2}}$$

$$a^2\left(1^2 + \tan^2\frac{\beta}{2}\right) = 1 \Rightarrow a^2\left(\frac{\cos^2\frac{\beta}{2} + \sin^2\frac{\beta}{2}}{\cos^2\frac{\beta}{2}}\right) = 1$$

$$\Rightarrow a = \cos\frac{\beta}{2}$$

$$\Rightarrow b = a e^{i\alpha}\left(\tan\frac{\beta}{2}\right) = \cos\frac{\beta}{2} e^{i\alpha}\tan\frac{\beta}{2} = e^{i\alpha}\sin\frac{\beta}{2}$$

$$\Rightarrow |\vec{S}\cdot\hat{n}\rangle = a|+\rangle + b|-\rangle = \cos\frac{\beta}{2}|+\rangle + e^{i\alpha}\sin\frac{\beta}{2}|-\rangle$$