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1.7 $|\alpha\rangle = \sum_{a'} |a'\rangle\langle a'|\alpha\rangle$

$$B = \prod_{a'} (A - a') = (A-a')(A-a'')(A-a''')\ldots$$

$$B|\alpha\rangle = \sum_{a''}\prod_{a'} (A-a')|a''\rangle\langle a''|\alpha\rangle$$

$$= \sum_{a''}\prod_{a'}(a''-a')|a''\rangle\langle a''|\alpha\rangle$$

so clearly every term is zero!

1.8 $S_x = \frac{\hbar}{2}\left[(|+\rangle\langle -|) + (|-\rangle\langle +|)\right]$

$$S_y = \frac{\hbar}{2}\left[-i(|+\rangle\langle -|) + i(|-\rangle\langle +|)\right]$$

$$S_z = \frac{\hbar}{2}\left[(|+\rangle\langle +|) - (|-\rangle\langle -|)\right]$$

$$S_x S_y = \frac{\hbar^2}{4}\left[|+\rangle\langle -| + (|-\rangle\langle +|)\right]\left[-i(|+\rangle\langle -|) + i(|-\rangle\langle +|)\right]$$

$$= \frac{\hbar^2}{4} i\, |+\rangle\langle +| - i\frac{\hbar^2}{4}|-\rangle\langle -| = i S_z \frac{\hbar}{2}$$

$$S_y S_x = -\frac{\hbar^2}{4} i(|+\rangle\langle +|) + \frac{\hbar^2}{4} i(|-\rangle\langle -|) = -i S_z \frac{\hbar}{2}$$

$\Rightarrow S_x S_y - S_y S_x = [S_x, S_y] = S_z \hbar \Rightarrow [S_y, S_x] = -S_z \hbar$

$$\{S_x, S_y\} = 0 = \{S_y, S_x\}$$

$$S_y S_z = \left(\frac{\hbar}{2}\right)^2 i\left[-|+\rangle\langle -| + |-\rangle\langle +|\right]\left[|+\rangle\langle +| - |-\rangle\langle -|\right] = \left(\frac{\hbar}{2}\right)^2 i\left[|+\rangle\langle -| + |-\rangle\langle +|\right]$$

$$= i\frac{\hbar}{2} S_x$$

$$S_z S_y = \left(\frac{\hbar}{2}\right)^2 i\left[-|+\rangle\langle -| - |-\rangle\langle +|\right) = -i\frac{\hbar}{2} S_x$$

$$[S_y, S_z] = i\hbar S_x \Rightarrow [S_z, S_y] = -i\hbar S_x$$

$$\{S_y, S_z\} = 0 = \{S_z, S_y\}$$

$$S_z S_x = \left(\frac{\hbar}{2}\right)^2 \left[|+\rangle\langle+| - |-\rangle\langle-|\right]\left[|+\rangle\langle-| + |-\rangle\langle+|\right]$$

$$= \left(\frac{\hbar}{2}\right)^2 \left[|+\rangle\langle-| - |-\rangle\langle+|\right] = -i^2 \left(\frac{\hbar}{2}\right)^2 \left[|+\rangle\langle-| - |-\rangle\langle+|\right]$$

$$= i\left(\frac{\hbar}{2}\right)\frac{\hbar}{2}\left[-i|+\rangle\langle-| + i|-\rangle\langle+|\right] = i\frac{\hbar}{2} S_y$$

$$S_x S_z = \left(\frac{\hbar^2}{2}\right)\left[-|+\rangle\langle-| + |-\rangle\langle+|\right] = -i^2\left(\frac{\hbar}{2}\right)^2\left[-|+\rangle\langle-| + |-\rangle\langle+|\right]$$

$$= -i\frac{\hbar}{2} S_y$$

$$[S_z, S_x] = i\hbar S_y \Rightarrow [S_x, S_z] = -i\hbar S_y$$

$$\{S_z, S_x\} = 0 = \{S_x, S_z\}$$

$$S_x S_x = \left(\frac{\hbar}{2}\right)^2 \left[|+\rangle\langle+| + |-\rangle\langle-|\right] = \left(\frac{\hbar}{2}\right)^2 \mathbb{1}$$

$$S_y S_y = \left(\frac{i\hbar}{2}\right)^2 \left[-|+\rangle\langle+| - |-\rangle\langle-|\right] = \left(\frac{\hbar}{2}\right)^2 \left[|+\rangle\langle+| + |-\rangle\langle-|\right] = \left(\frac{\hbar}{2}\right)^2 \mathbb{1}$$

$$S_z S_z = \left(\frac{\hbar}{2}\right)^2 \left[|+\rangle\langle+| + |-\rangle\langle-|\right] = \left(\frac{\hbar}{2}\right)^2 \mathbb{1}$$

obviously $[A, A] = 0$ for any operator, so

$$[S_x, S_x] = [S_y, S_y] = [S_z, S_z] = 0$$

and

$$\{S_x, S_x\} = \{S_y, S_y\} = \{S_z, S_z\} = \left(\frac{\hbar}{2}\right)^2 \mathbb{1}$$

So from these we conclude

$$[S_i, S_j] = i\varepsilon_{ijk}\hbar S_k \quad \text{and} \quad \{S_i, S_j\} = \left(\frac{\hbar}{2}\right)^2 \delta_{ij} \mathbb{1}$$