Vickle, Zachary | QM Sakurai | 08/27/18 | 1/1

1.5 a) $|\alpha\rangle = |a'\rangle\langle a'|\alpha\rangle + |a''\rangle\langle a''|\alpha\rangle + \ldots = \sum_{a'} |a'\rangle\langle a'|\alpha\rangle$

$|\beta\rangle = |a'\rangle\langle a'|\beta\rangle + |b''\rangle\langle b''|\beta\rangle + \ldots = \sum_{a'} |a'\rangle\langle a'|\beta\rangle$

$\langle\beta| = \langle\beta|a'\rangle\langle a'| + \langle\beta|a''\rangle\langle a''| + \ldots = \sum_{a'} \langle\beta|a'\rangle\langle a'|$

$$|\alpha\rangle\langle\beta| = |a'\rangle\langle a'|\alpha\rangle\langle\beta|a'\rangle\langle a'| + |a'\rangle\langle a'|\alpha\rangle\langle\beta|a''\rangle\langle a''| + \ldots$$
$$+ |a''\rangle\langle a''|\alpha\langle\beta|a'\rangle\langle a'| + |a''\rangle\langle a''|\alpha\rangle\langle\beta|a''\rangle\langle a''| + \ldots$$

$$= \sum_{a''} |a'\rangle\langle a'|\alpha\rangle\langle\beta|a''\rangle\langle a''|$$
$$+ \sum_{a'} |a''\rangle\langle a''|\alpha\rangle\langle\beta|a'\rangle\langle a'|$$
$$+ \ldots$$

$$= \sum_{a'}\sum_{a''} |a'\rangle\langle a'|\alpha\rangle\langle\beta|a''\rangle\langle a''|$$

Consequently, we simply need to multiply $|\alpha\rangle\langle\beta|$ by the identity operator on the left and right (eq. 1.3.32)

$$|\alpha\rangle\langle\beta| = \sum_{a''}\sum_{a'} |a''\rangle\langle a''|\alpha\rangle\langle\beta|a'\rangle\langle a'|$$

b)

$|\beta\rangle = |S_x;+\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}} e^{i\delta_1}|-\rangle$

$|\alpha\rangle = |S_z;+\rangle = |+\rangle$

$$|\alpha\rangle\langle\beta| = \frac{1}{\sqrt{2}}|+\rangle\langle +| + \frac{1}{\sqrt{2}} e^{-i\delta_1}|+\rangle\langle -|$$